High-Precision Branching Ratio Measurement for the Superallowed Fermi β Emitter $^{22}\text{Mg}$

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Beta Decay and $ft$ Values

From Fermi’s Golden Rule:

$$ft = \frac{K}{g^2 |M_{fi}|^2}$$

Experimentally determine:
- $Q$ value
- Half-life ($T_{1/2}$)
- Branching Ratio (BR)

$g = G_V + G_A$

(V–A Theory)
$f t$ Values for Superallowed Transitions

In the special case of pure Fermi transitions between isobaric analogue states, we have that:

1. $g = G_V$ (constant from CVC)
2. $|M_{fi}|^2 = (T - T_Z)(T + T_Z + 1) = 2$ (isospin ladder operator)

From Fermi’s Golden Rule:

$$f t = \frac{K}{g^2 |M_{fi}|^2}$$

$$f t = \frac{2\pi^3 \hbar^7 \ln 2}{2G^2 V m_e^5 c^4} = \text{constant}$$
$f_t$ Values for Superallowed Transitions

In the special case of pure Fermi transitions between isobaric analogue states, we have that:

1. \[ g = G_V \text{ (constant from CVC)} \]
2. \[ |M_{fi}|^2 = (T-T_Z)(T+T_Z+1) = 2 \text{ (isospin ladder operator)} \]

\[
\frac{f_t}{c} = \frac{2\pi^3 \hbar^7 \ln 2}{2G_V^2 m_e c^4} = \text{constant}
\]

[Graph showing $f_t$ values for different isotopes, with a trend indicating constancy within 1.5% error.]
Isospin Symmetry Breaking

Superallowed Fermi $\beta$ decays occur between nuclear isobaric analogue states.

A proton is converted into a neutron with an (almost) identical wavefunction, so that the transition matrix element is trivial: $\langle f | O | i \rangle = \sqrt{2}$ (for $T = 1$).

The exact symmetry between proton and neutron wavefunctions in the nucleus is broken by Coulomb and charge-dependent nuclear interactions.
Superallowed Fermi $\beta$ Decay: Corrections

$$\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G^2_V (1 + \Delta^V_R)} = \text{constant}$$

- **Corrected $ft$ value**
- **Calculated corrections (~1%)** (nucleus dependent)
- **Inner radiative correction (~2.4%)** (nucleus independent)
- **CVC Hypothesis**

\[ \Delta^V_R = \text{nucleus independent inner radiative correction: } 2.361(38)\% \]
\[ \delta'_R = \text{nucleus dependent radiative correction to order } Z^2\alpha^3: \sim 1.4\% \]
\[ \delta'_R \text{ depends on electron’s energy and } Z \text{ of nucleus} \]
\[ \delta_{NS} = \text{nuclear structure dependent radiative correction: } -0.3\% – 0.03\% \]
\[ \delta_C = \text{nucleus dependent isospin-symmetry-breaking correction: } 0.2\% – 1.5\% \]
\[ \delta_C \text{ strong nuclear structure dependence} \]
Corrected Superallowed $\mathcal{F}t$ Values

$\mathcal{F}t = \frac{K}{2 G_V^2}$

Confirmation of CVC!

and

$|V_{ud}| = \frac{G_V}{G_F}$

Fermi coupling constant (from leptonic decays)

$\mathcal{F}t_{WS} = 3072.27(62)_{\text{stat}}(36)_{\delta_R} \text{ s, } \chi^2/\nu = 0.52$
Impacts of studying $T=1$ superallowed Fermi $\beta$ Emitters

Test of conserved vector current (CVC) hypothesis
confirmed to better than 12 parts in $10^5$

Most precise determination of $V_{ud}$

$|V_{ud}| = 0.97420(21)$

Tests of CKM unitarity

$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = 0.00038(49)$

World survey consists of some 220 individual measurements

Search for physics beyond the Standard Model

Fundamental or induced scalar currents in Weak interaction
Corrected $ft$ values & $^{22}\text{Mg}$

$$\mathcal{F}t = ft(1 + \delta'_{R})(1 + \delta_{NS} - \delta_{C}) = \frac{K}{2G_{V}^2(1 + \Delta_{V}^R)} = \text{constant}$$

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Corrected $ft$ values & $^{22}\text{Mg}$

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Corrected $ft$ values & $^{22}$Mg

$$\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta^V_R)} = \text{constant}$$
Status of $^{22}\text{Mg} f_t$ Value

$$f_t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$$
Status of $^{22}$Mg $ft$ Value

Fractional Uncertainty (%)

**Goal:** To improve the $^{22}$Mg branching ratio measurement to the $\pm 0.15\%$ level

$$ft = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$$
As decay to the ground state is 2\textsuperscript{nd} forbidden, the superallowed branching ratio can be determined through the measurement of relative $\gamma$ ray intensities:

$$\text{BR(sa)} = \frac{I_\gamma(74)(1+\alpha) - I_\gamma(1280)}{I_\gamma(583) + I_\gamma(1937)}$$

with $\alpha(74) = 0.00357(5)$. 

\textbf{$^{22}\text{Mg}$ Superallowed Branching Ratio}
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$I(\gamma) = \frac{N(\text{observed})}{\epsilon_{\gamma}}$

$$\approx \frac{N_{\gamma}(74)\epsilon_{\gamma}(583) - N_{\gamma}(1280)\epsilon_{\gamma}(583)}{N_{\gamma}(583)\epsilon_{\gamma}(74) - N_{\gamma}(583)\epsilon_{\gamma}(1280)}$$

$I_{\gamma}(74)$ and $I_{\gamma}(583)$ are relative gamma-ray intensities.
\(^{22}\text{Mg}\) Superallowed Branching Ratio

\[
\text{BR}(\text{sa}) \approx \frac{N_\gamma(74)\varepsilon_\gamma(583)}{N_\gamma(583)\varepsilon_\gamma(74)} - \frac{N_\gamma(1280)\varepsilon_\gamma(583)}{N_\gamma(583)\varepsilon_\gamma(1280)}
\]

With a precisely calibrated HPGe detector, Hardy et al., PRL 91, 092501 (2003) obtained:

\[
\text{BR} = 53.15(12)\% \ (\pm 0.23\%)
\]

J.C. Hardy et al., PRL 91, 082501 (2003)

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With a high $\gamma$--$\gamma$ efficiency, one can establish the relative efficiency between 74 and 583 keV \textit{in situ} by gating on the 1280 keV $\gamma$ ray.

To measure the BR to $\pm 0.15\%$, we need to measure $\varepsilon_\gamma$ to $\pm 0.1\%$. 
GRiffin @ ISAC

A new high-efficiency decay spectroscopy facility for ISAC-I

- Comprised of 16 large-volume clover-type HPGe
- Tape transport system
- Ancillary detectors for beta tagging (+ CE + LaBr$_3$ + neutrons)
Experimental Results

- $^{22}\text{Mg}$ beam attenuated to $2.5 \times 10^4$/s to reduce pile-up
- $>2 \times 10^6$ counts in 1280-coincident 74 and 583 peaks means statistical precision of $\varepsilon_\gamma$ of 0.1% achieved
- Summing and pile-up corrections underway
Thank you for your attention

H. Bidaman; V. Bildstein; C. Burbadge; A. Diaz Varela; M. R. Dunlop; R. Dunlop; P. E. Garrett; B. Jigmeddorj; A. D. MacLean; C. E. Svensson; J. Turko; T. Zidar

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