Magnetometry and magnetic-field decomposition for the TUCAN nEDM experiment

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February 15, 2019
Neutron EDM

- EDM describes the torque a particle experiences due to an external electric field

- In an external E and B field, the Hamiltonian is:
  \[ H = -\vec{\mu}_n \cdot \vec{B} - \vec{d}_n \cdot \vec{E} \]

- The vector dipole moment and the axial vector magnetic moment transform differently under various symmetries
Measuring the nEDM

- Rather than reverse time in the lab, we can have the electric and magnetic fields be either parallel or anti-parallel.
- A non-zero EDM results in a change in precession frequency.

\[ \omega_+ = \frac{(2\mu_n B + 2d_n E)}{\hbar} \]
\[ \omega_- = \frac{(2\mu_n B - 2d_n E)}{\hbar} \]
For a measurable $\Delta \omega$, neutrons must precess for a long time
   - $B_0$ field can drift
Another atomic species in the same volume can measure the magnetic field the entire time, correcting for drift

In terms of the scalar potential:

$$d_{f,\text{Hg}} = -\frac{\hbar \gamma^2}{2c^2} \langle xB_x + yB_y \rangle$$

$$d_{\text{false}} = -\frac{\hbar \gamma_n \gamma_{\text{Hg}}}{2c^2} \langle \rho \frac{\partial \Phi}{\partial \rho} \rangle$$
Correcting dFalse

• My goal: accurately characterize dFalse, a large correction to our measured nEDM

Place magnetometers inside the volume

Infer field inside cell by measuring field outside

Passive magnetic shielding
Field decomposition

• In free space, magnetic scalar potential obeys Laplace’s equation:

\[ \nabla^2 \Phi = 0 \]

• General solution:

\[
\Phi = \sum_{l,m} [A_{l,m} r^l + B_{l,m} r^{-(l+1)}] [C \cos m\phi + D \sin m\phi] [E P_l^m(\cos \theta) + F Q_l^m(\cos \theta)]
\]

• A convenient choice that satisfies this solution is a renormalization of the real spherical harmonics\(^2\):

\[ \Phi_{l,m} = C_{l,m}(\phi) r^l P_l^{|m|}(\cos \theta) \]

• with

\[
C_{l,m}(\phi) = \frac{(l - 1)!(-2)^{|m|}}{(l + |m|)!} \cos(m\phi) \quad \text{for} \quad m \geq 0
\]

\[
= \frac{(l - 1)!(-2)^{|m|}}{(l + |m|)!} \sin(|m|\phi) \quad \text{for} \quad m < 0
\]

Field decomposition

- To arbitrary order, then

\[
\vec{B}(\vec{r}) = \sum_{l,m} G_{l,m} \nabla \Phi(\vec{r})
\]

\[
\begin{pmatrix}
B_x(\vec{r}) \\
B_y(\vec{r}) \\
B_z(\vec{r})
\end{pmatrix} = \sum_{l,m} G_{l,m} \begin{pmatrix}
\Pi_{x,l,m}(\vec{r}) \cdot \hat{i} \\
\Pi_{y,l,m}(\vec{r}) \cdot \hat{j} \\
\Pi_{z,l,m}(\vec{r}) \cdot \hat{k}
\end{pmatrix}
\]

These gradient values fully parameterize the magnetic field.
Field decomposition

- Our $B_0$ field oriented along the $z$ axis
- $B_z$ field independent of some $G$ terms
- Order of expansion = order of polynomial terms
- False EDM is written in terms of the gradients, so we need to extract $g$:

\[
\vec{g} = p\text{inv}(X_z) \ast \vec{B}_z
\]
False EDM

- False EDM calculated from the gradient terms $^3$:

$$d_{f,\text{Hg}} = -\frac{\hbar\gamma^2}{2c^2} \langle xB_x + yB_y \rangle = \frac{\hbar\gamma^2 R^2}{8c^2} \left[ g_{10} + g_{30} \left( \frac{H^2}{4} - \frac{R^2}{2} \right) + g_{50} \left( \frac{H^4}{16} - \frac{5R^2H^2}{12} + \frac{5R^4}{16} \right) \right]$$

- This is the quantity I want to accurately extract for any given magnetic field

Positions

- Working equation must be adequately constrained – e.g. can’t have all sensors on equi-magnetic surface
- There must be some set of positions that performs the best, but a 3\textsuperscript{rd} order fit requires $16 \times 3 = 48$ position parameters to optimize
- Solution $\rightarrow$ have a computer decide for me
- A genetic algorithm can do this
Using a GA

- Set # of positions
- Set sensing error
- Set offset error
- Set placement error
- Set volume of Cs cell

Have Ferret find optimal positions

Test system on 100,000 random gradients

Does the system perform well enough?

Make sensors better

No

Yes

Make sensors easier to build

4) Ferret is part of the nQube software package developed by Dr. Jason Fiege at the University of Manitoba
Optimization results

- **Testing 10,000 gradients:**
  - Field sensing error: 10 fT
  - Position error: 0.1 cm
  - # sensors: 17(+1)

Below you can see the error is always smaller than 3*e-28!
Atomic magnetometry

- Certain polarization states of alkali atoms behave very predictably in magnetic fields, atomic (or optical) magnetometry exploits this.

- Measures laser polarization rotation in response to B fields.

- Moushumi Das demonstrated a working Rb based magnetometer with a statistical sensitivity of 0.1 pT over 10 seconds, setup diagram to the right.

Above: Working Cs magnetometers at the PSI nEDM experiment, taken from S. Komposch’s Ph.D thesis.
Conclusion

- **dFalse** is the dominant source of systematic uncertainty in the experiment. By placing Cs magnetometers in the experimental volume we can theoretically characterize this false signal to a precision of order $1\times10^{-28}$ e·cm

- Future work: in order to operate these magnetometers in the experimental volume, the laser light will be provided via fibre optic cables, prelim. design below.

A preliminary design for the magnetometers. They need to be made with zero metal, to avoid contaminating the magnetic field in the volume. This makes precision alignment difficult.