# Magnetometry and magnetic-field decomposition for the TUCAN nEDM experiment

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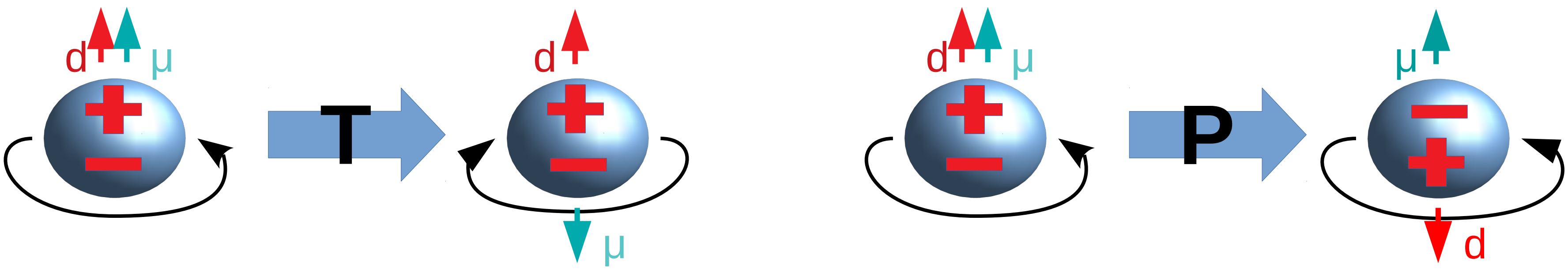
# Neutron EDM

- EDM describes the torque a particle experiences due to an external electric field
- In an external E and B field, the Hamiltonian is:

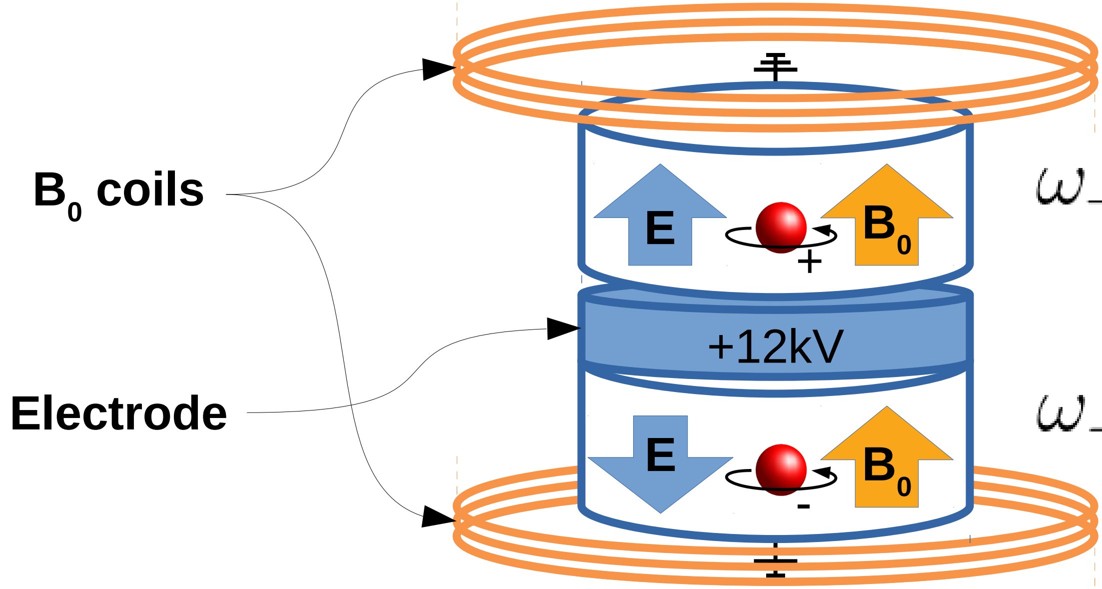
$$H = -\vec{\mu_n} \cdot \vec{B} - \vec{d_n} \cdot \vec{E}$$

- The vector dipole moment and the axial vector magnetic moment transform differently under various symmetries



# Measuring the nEDM

- Rather than reverse time in the lab, we can have the electric and magnetic fields be either parallel or anti-parallel
- A non-zero EDM results in a change in precession frequency

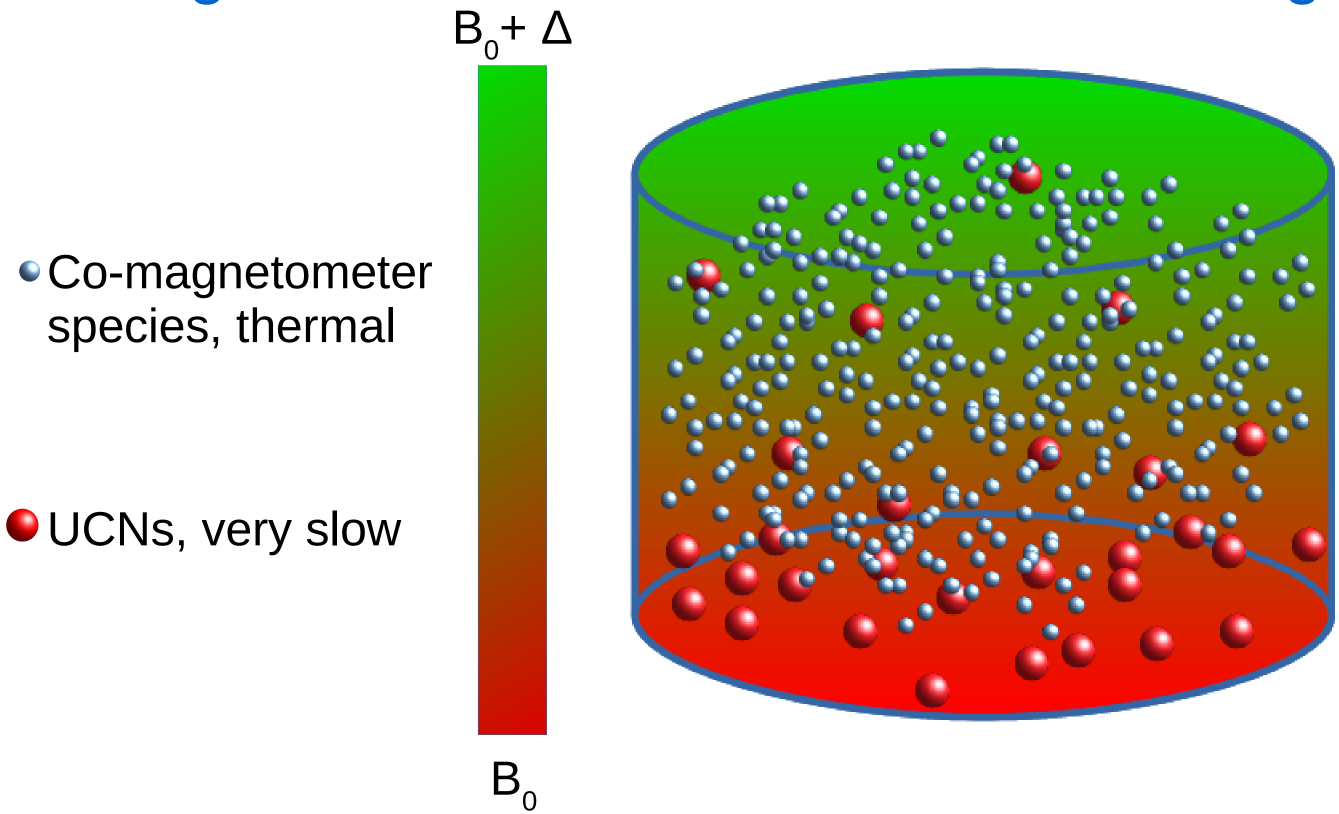



$$\omega_+ = (2\mu_n B + 2d_n E)/\hbar$$

$$\omega_- = (2\mu_n B - 2d_n E)/\hbar$$

# $B_0$ homogeneity

- For a measurable $\Delta\omega$, neutrons must precess for a long time
  - $B_0$ field can drift
- Another atomic species in the same volume can measure the magnetic field the entire time, correcting for drift




**False EDM** signal[1]

$$d_{f,\mathrm{Hg}} = -\frac{\hbar\gamma^2}{2c^2}\langle xB_x + yB_y\rangle$$

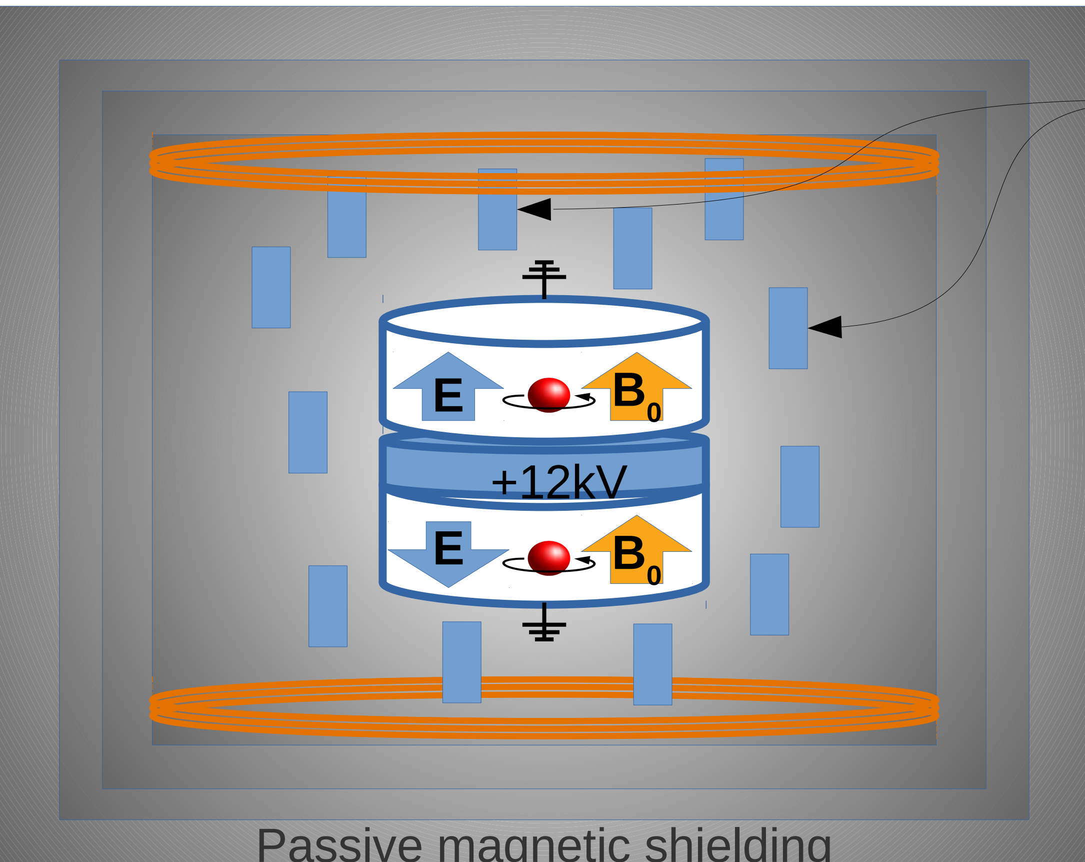
In terms of the scalar potential:

$$d^{\mathrm{false}}_{n\leftarrow\mathrm{Hg}} = -\frac{\hbar\gamma_n\gamma_{\mathrm{Hg}}}{2c^2}\langle\rho\frac{\partial\Phi}{\partial\rho}\rangle$$

1) G. Pignol, S. Roccia, Electric dipole moment searches: Reexamination of frequency shifts for particles in traps, Phys. Rev. A85 (2012) 042105.

# Correcting dFalse

- My goal: accurately characterize dFalse, a large correction to our measured nEDM




- Place magnetometers inside the volume
- Infer field inside cell by measuring field outside

# Field decomposition

- In free space, magnetic scalar potential obeys Laplace's equation:

$$\nabla^2\Phi = 0$$

- General solution:

$$\Phi = \sum_{l,m}[A_{l,m}r^l + B_{l,m}r^{-(l+1)}][C\cos m\phi + D\sin m\phi][EP_l^m(\cos\theta) + FQ_l^m(\cos\theta)]$$

- A convenient choice that satisfies this solution is a re-normalization of the real spherical harmonics[2]:

$$\Phi_{l,m} = C_{l,m}(\phi)r^l P_l^{|m|}(\cos\theta)$$

- with

$$\begin{aligned} C_{l,m}(\phi) &= \frac{(l-1)!(-2)^{|m|}}{(l+|m|)!}\cos(m\phi) && \text{for} \quad m \geq 0 \\ &= \frac{(l-1)!(-2)^{|m|}}{(l+|m|)!}\sin(|m|\phi) && \text{for} \quad m < 0 \end{aligned}$$

2) C. Abel, et al., Magnetic field uniformity in neutron electric dipole moment experiments, arXiv:1811.06085 [physics.ins-det], 2018.

# Field decomposition

- To arbitrary order, then

$$\vec{B}(\vec{r}) = \sum_{l,m} G_{l,m} \overbrace{\nabla\Phi(\vec{r})}^{\Pi_{x_i,l,m}}$$

$$\begin{pmatrix} B_x(\vec{r}) \\ B_y(\vec{r}) \\ B_z(\vec{r}) \end{pmatrix} = \sum_{l,m} G_{l,m} \begin{pmatrix} \Pi_{x,l,m}(\vec{r}) \cdot \hat{i} \\ \Pi_{y,l,m}(\vec{r}) \cdot \hat{j} \\ \Pi_{z,l,m}(\vec{r}) \cdot \hat{k} \end{pmatrix}$$

These gradient values fully parameterize the magnetic field

$$\begin{bmatrix} B_x(x_1,y_1,z_1) \\ \vdots \\ B_y(x_1,y_1,z_1) \\ \vdots \\ B_z(x_1,y_1,z_1) \\ \vdots \\ B_z(x_n,y_n,z_n) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & y_1 & 0 & \frac{-1}{2}x_1 & z_1 & x_1 & 2x_1y_1 & 2y_1z_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ 1 & 0 & 0 & x_1 & z_1 & \frac{-1}{2}y_1 & 0 & -y_1 & x_1^2 - y_1^2 & 2x_1z_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ 0 & 1 & 0 & 0 & y_1 & z_1 & x_1 & 0 & 0 & 2x_1y_1 \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ 0 & 1 & 0 & 0 & y_n & z_n & x_n & 0 & 0 & 2x_ny_n \cdots \end{bmatrix} \cdot \begin{bmatrix} G_{0-1} \\ G_{00} \\ G_{01} \\ G_{1-2} \\ G_{1-1} \\ G_{10} \\ G_{11} \\ G_{12} \\ G_{2-2} \\ G_{2-1} \\ \vdots \end{bmatrix}$$

$\vec{B}_z$ $X_z$ $\vec{g}$

# Field decomposition

- Our $B_0$ field oriented along the z axis

$$\begin{bmatrix} B_z(x_1, y_1, z_1) \\ \vdots \\ B_z(x_n, y_n, z_n) \end{bmatrix} = \begin{bmatrix} \cancel{0} & 1 & \cancel{0} & \cancel{0} & y_1 & z_1 & x_1 & \cancel{0} & \cancel{0} & 2x_1 y_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & & \\ \cancel{0} & 1 & \cancel{0} & \cancel{0} & y_n & z_n & x_n & \cancel{0} & \cancel{0} & 2x_n y_n & \cdots \end{bmatrix} \cdot \begin{bmatrix} \cancel{G_{0-1}} \\ G_{00} \\ \cancel{G_{01}} \\ \cancel{G_{1-2}} \\ G_{1-1} \\ G_{10} \\ G_{11} \\ \cancel{G_{12}} \\ \cancel{G_{2-2}} \\ G_{2-1} \\ \vdots \end{bmatrix}$$

- $B_z$ field independent of some G terms
- Order of expansion = order of polynomial terms
- False EDM is written in terms of the gradients, so we need to extract g:

$$\vec{g} = pinv(X_z) * \vec{B}_z$$

Working equation

# False EDM

- False EDM calculated from the gradient terms[3]:

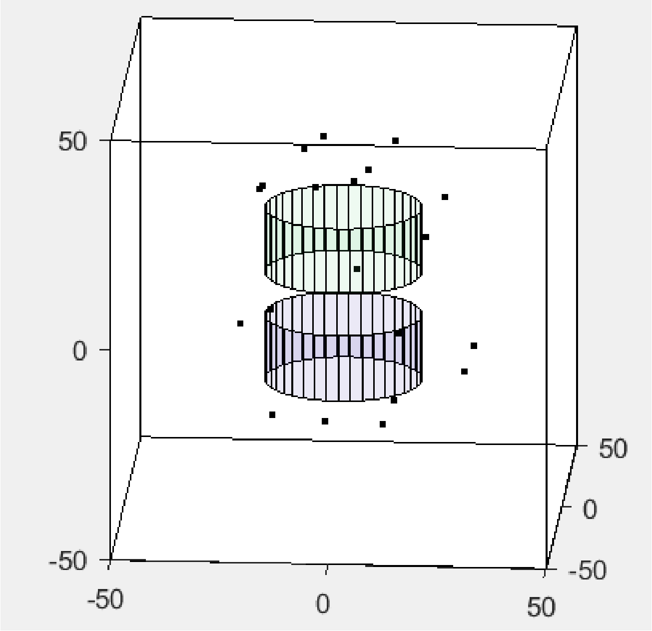
$$d_{f,\mathrm{Hg}} = -\frac{\hbar\gamma^2}{2c^2}\langle xB_x + yB_y\rangle = \frac{\hbar\gamma^2 R^2}{8c^2}\left[g_{10} + g_{30}\left(\frac{H^2}{4} - \frac{R^2}{2}\right) + g_{50}\left(\frac{H^4}{16} - \frac{5R^2H^2}{12} + \frac{5R^4}{16}\right)\right]$$

- This is the quantity I want to accurately extract for any given magnetic field

3) Klassen, W. et al., (2019). *Wigner–Eckart theorem and the false EDM of 199Hg*. NIM-A, 922, 322-325. doi:10.1016/j.nima.2019.01.019

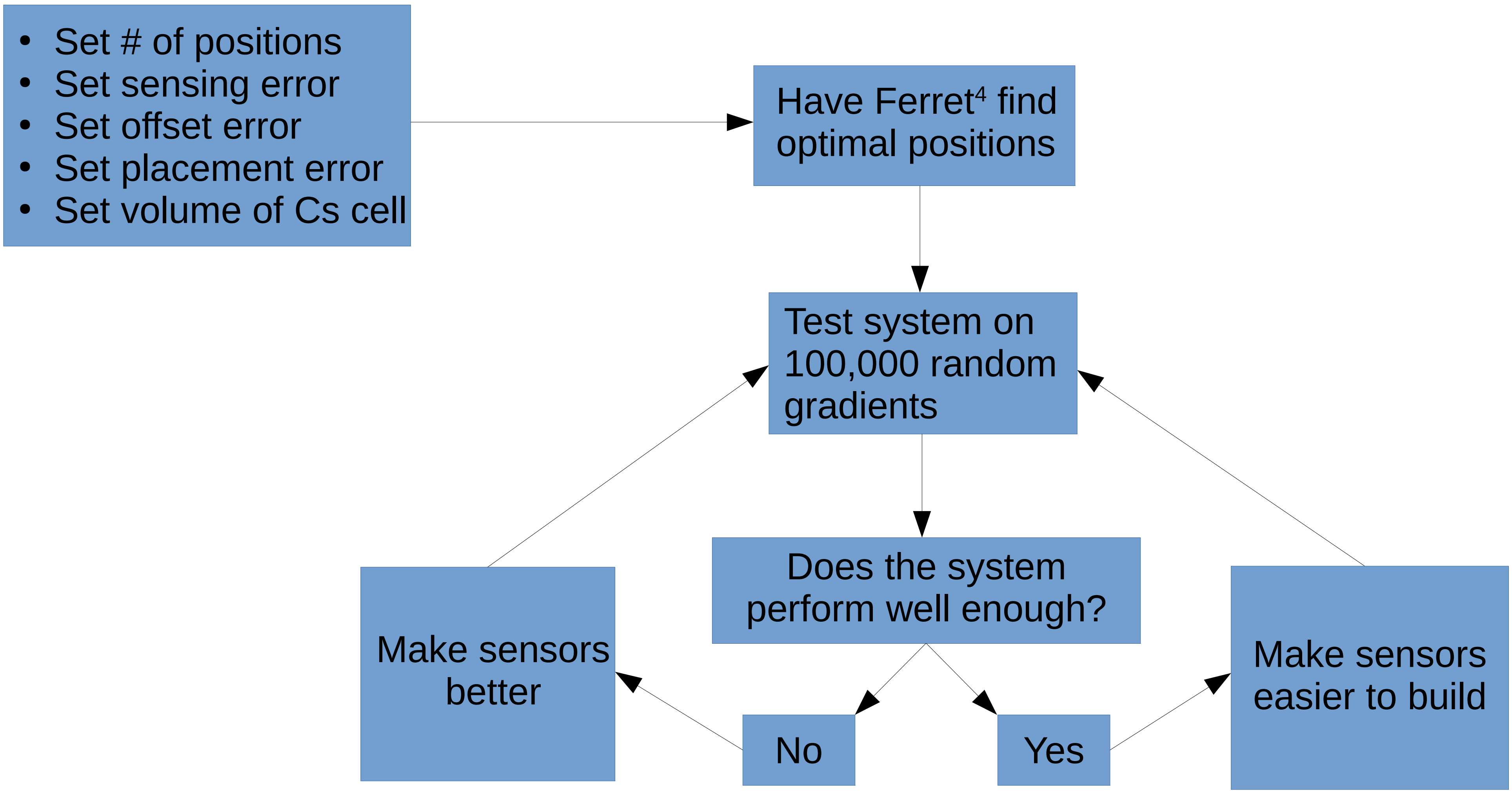
# Positions

- Working equation must be adequately constrained – e.g. can't have all sensors on equi-magnetic surface
- There must be some set of positions that performs the best, but a 3rd order fit requires 16*3 = 48 position parameters to optimize



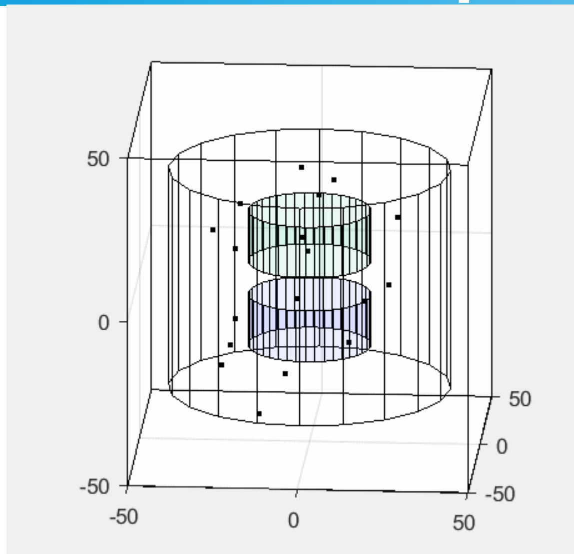

- Solution → have a computer decide for me
- A genetic algorithm can do this

# Using a GA




4) Ferret is part of the nQube software package developed by Dr. Jason Fiege at the University of Manitoba

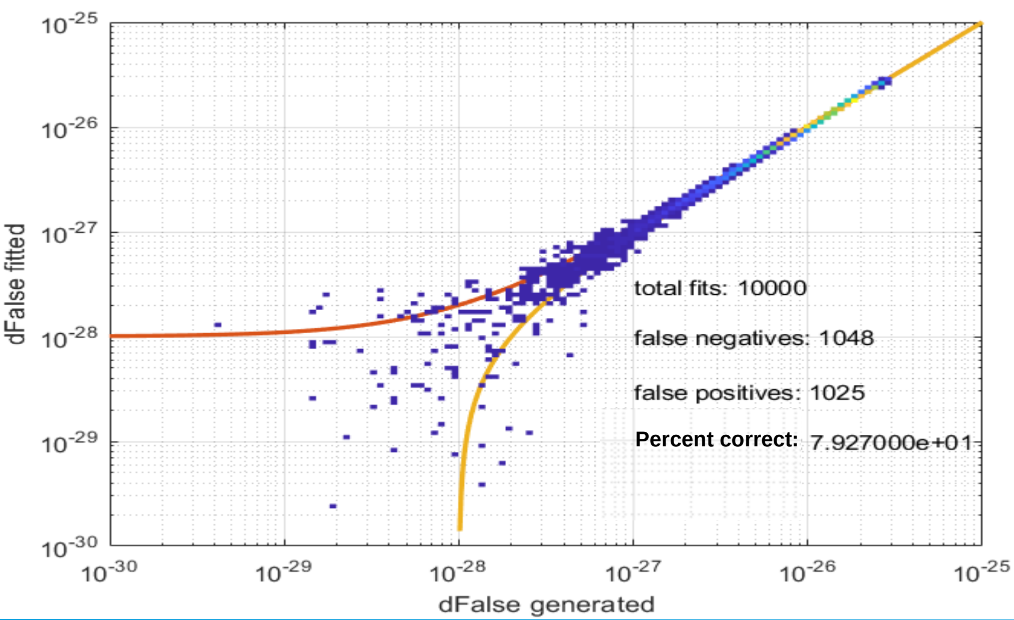
# Optimization results

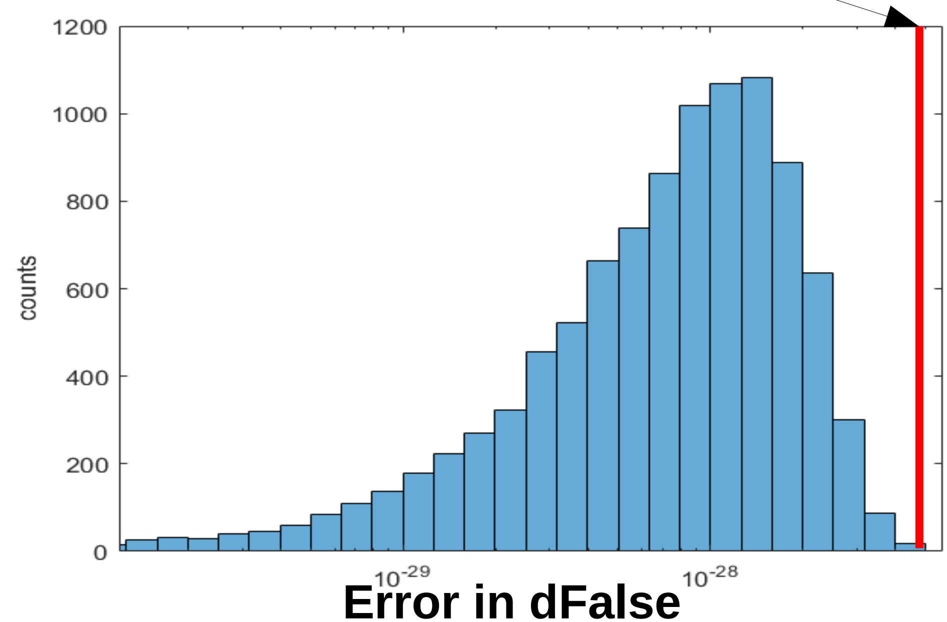



- Testing 10,000 gradients:

Field sensing error: 10 fT
Position error: 0.1 cm
# sensors: 17(+1)

Below you can see the error is always smaller than 3*e-28!



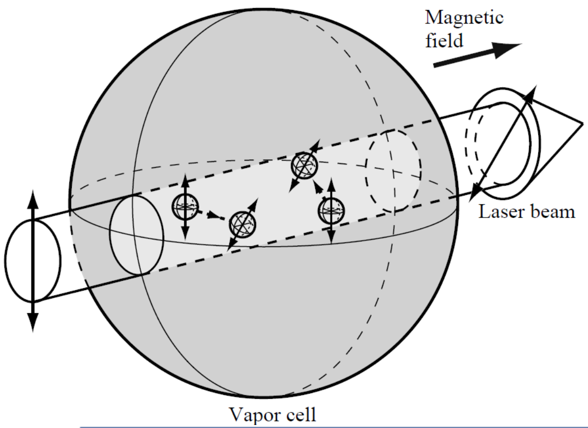



# Atomic magnetometry

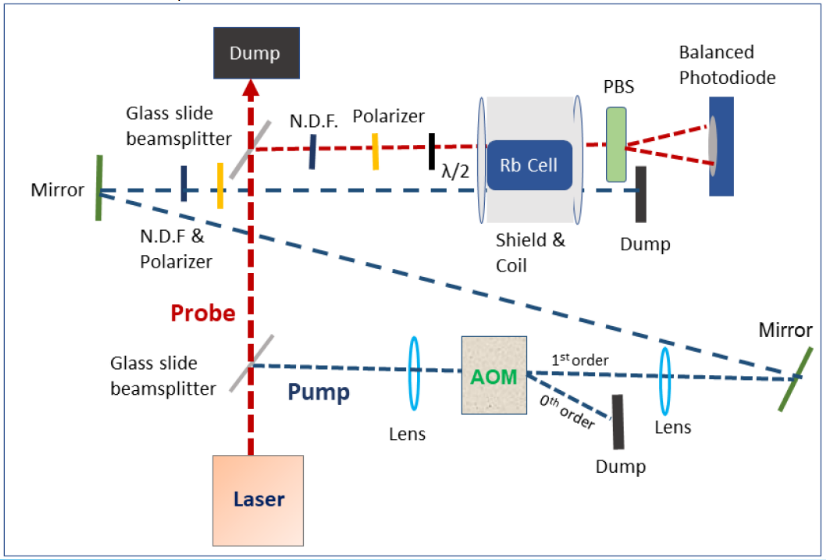
- Certain polarization states of alkali atoms behave very predictably in magnetic fields, atomic (or optical) magnetometry exploits this

- Measures laser polarization rotation in response to B fields

- Moushumi Das demonstrated a working Rb based magnetometer with a statistical sensitivity of 0.1 pT over 10 seconds, setup diagram to the right

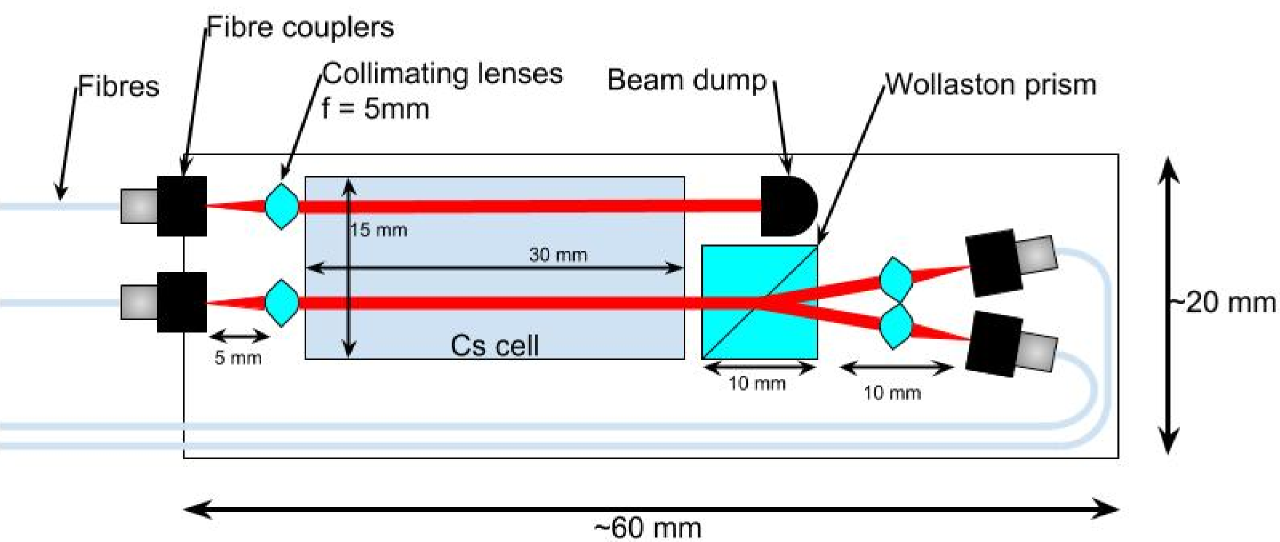



Left: figure taken from Optical Magnetometry by Budker & Kimball



# Conclusion

- **dFalse** is the dominant source of systematic uncertainty in the experiment.  By placing Cs magnetometers in the experimental volume we can theoretically characterize this false signal to a precision of order 1*e-28 e·cm
- Future work: in order to operate these magnetometers in the experimental volume, the laser light will be provided via fibre optic cables, prelim. design below




A preliminary design for the mag-heads.  They  need to be made with zero metal, to avoid contaminating the magnetic field in the volume.  This makes precision alignment difficult.

END