Next-to-Leading Order Dilepton Production Calculations

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Feb 15, 2019

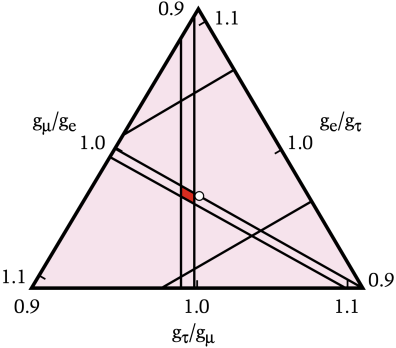
Outline

Dilepton Production

- Motivation
- Introduction
- Loop Calculation
- Results

Motivation

The motivation for it is to study lepton universality, which means all leptons behave similarly throughout generations.

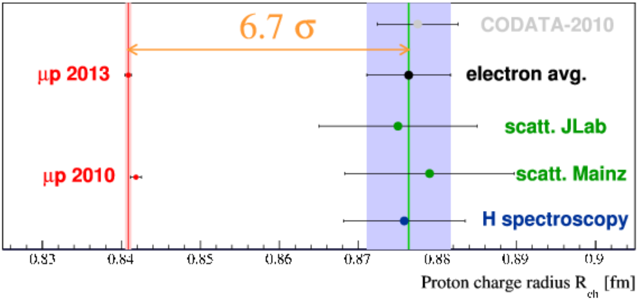


J. Ritchie Patterson, LEPTON UNIVERSALITY.

http://www.slac.stanford.edu/pubs/beamline/25/1/25-1-patterson.pdf

Motivation

Proton radius problem: Electron scattering experiments measure the proton radius as $r_e = 0.879(8)$ fm, while the muonic measurements read $r_\mu = 0.84087(39)$ fm . There are two main directions for this for the proton radius puzzle. One is looking for systematic measurement error. The other one is looking for physics beyond the Standard Model.



J. Arrington, ''An examination of proton charge radius extractions from e-p scattering data,'' J. Phys. Chem. Ref Data **44**, 031203 (2015)

Dilepton Production

The scattering channel of my thesis research is

$$\gamma + p \rightarrow l^+ + l^- + p,$$

where the l^+ and l^- is the dilepton pairs produced by the scattering.

By studying the difference between the electron and muon pair production, we can obtain more information about their coupling constants and the lepton universality violation.

Scattering Channel

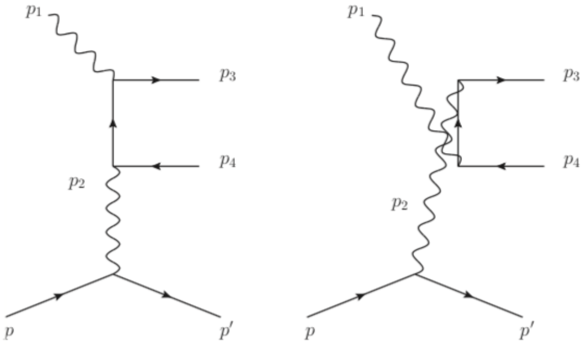


Figure: Feynman diagrams of tree level Bethe-Heitler process

M. Heller, O. Tomalak, and M. Vanderhaeghen. Softphoton corrections to the Bethe-Heitler process in the γ p to l+l-p reaction. Phys. Rev., D97(7): 076012, 2018. doi: 10.1103/PhysRevD.97.076012.

Introduction

Applying Feynman rules, one can obtain the amplitude as

$$\begin{aligned}\mathscr{M}_0 =&\bar{u}(p3)(ie)\left[\gamma^\nu \frac{i(\not{p}_3 - \not{p}_1 + m)}{(p_3-p_1)^2 - m^2}\gamma^\mu + \gamma^\mu \frac{i(\not{p}_1 - \not{p}_4 + m)}{(p_1-p_4)^2 - m^2}\gamma^\nu\right](ie)v(p_4)\times \\ &\times \frac{-i}{t}\epsilon_\nu(p_1)\bar{u}(p')(-ie)\Gamma_\mu(t)u(p),\end{aligned} \tag{1}$$

where

$$\Gamma^\mu(t) = F_D(t)\gamma^\mu - iF_P(t)\frac{\sigma^{\mu\nu}(p_2)_\nu}{2M},$$

$$p_1^2 = 0,$$

$$p_3^2 = p_4^2 = m^2,$$

$$p^2 = p'^2 = M^2.$$

F_D and F_P is the Dirac and Pauli form factors of the photon.

Cross Section

And the corresponding differential cross section is

$$\frac{d\sigma}{dtds_{ll}d\Omega_{ll}^{CM}} = \frac{1}{(2\pi)^4}\frac{1}{64}\frac{\beta}{2ME_\gamma}\sum_{i,f}|\mathscr{M}_0|^2, \tag{2}$$

where

$$(p_3+p_4)^2 = s_{ll}, \;\; (p_3-p_1)^2 = t_{ll},$$
$$p_2^2 = (p-p')^2 = t.$$

These are known as Mandelstam variables (Lorentz invariant variables). and

$$\beta = \sqrt{1-\frac{4m^2}{s_{ll}}}$$

is the velocity of the lepton and E_γ is the lab energy of the initial photon and Ω_{ll} is the solid angle of the ll pair in the center of mass frame.

Cross Section

By averaging over initial polarization states and summing over final polarization states the differential cross section becomes

$$\frac{d\sigma}{dtds_{ll}d\Omega_{ll}^{CM}} = \frac{\alpha^3\beta}{16\pi(2ME_\gamma)^2t^2}L_{\mu\nu}H^{\mu\nu}, \tag{3}$$

where α is the fine structure constant $\alpha \equiv \frac{e^2}{4\pi} \approx \frac{1}{137}$,
and $L_{\mu\nu}$ is known as the leptonic tensor and $H^{\mu\nu}$ is known as the hadronic tensor.

Hadronic Tensor

Hadronic tensor has the form of

$$H^{\mu\nu} = \frac{1}{2} Tr[(\not{p}' + M)\Gamma^{\mu}(\not{p} + M)(\Gamma^{\dagger})^{\nu}].$$

The unpolarized hadronic tensor can be further expand into

$$H^{\mu\nu} = (-g^{\mu\nu} + \frac{p_2^{\mu} p_2^{\nu}}{p_2^2})[4M^2 \tau G_M^2(t)] + \tilde{p}^{\mu} \tilde{p}^{\nu} \frac{4}{1+\tau}[G_E^2(t) + \tau G_M^2(t)], \quad (4)$$

with $\tilde{p} \equiv (p + p')/2$ and $\tau \equiv -t/(4M^2)$ and G_M (Magnetic Sachs form factor) and G_E (Electric Sachs form factor) are in the form of

$$G_M = F_D + F_P$$

$$G_E = F_D - \tau F_P$$

Leptonic Tensor

Thanks to the separation of hadronic tensor, we can cut the hadronic current out of the Bethe-Heitler process to calculate the leptonic tensor alone. Due to the high precision requirement of the experiment, one needs to include higher order diagrams into the cross section calculations. The next to the leading order level or one loop level provides a better approximation.

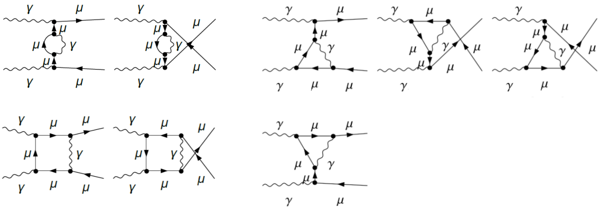


Figure: Feynman diagrams for one-loop level leptonic current

On-Shell Renormalization Conditions

The general idea of the renormalization scheme is to develop counterterms derived from the Lagrangian, which remove the divergence in the amplitude. The renormalized result of $\Sigma(k)$ is shown in the form of $\hat{\Sigma}(k)$.
Renormalization conditions are used to derived renormalization constants. On-shell renormalization conditions require the kinematic condition on-shell $k^2 = m^2$.

1. For photon-photon self energy:

$$\left.\frac{\partial \hat{\Sigma}_{\gamma\gamma}}{\partial k^2}\right|_{k^2=0} = 0; \tag{5}$$

2. The condition also includes the vertex Thomson limit as the non-relativistic charge:

$$\left.\hat{\Gamma}^{\gamma ff}_{\mu,tot}(q^2)\right|_{q^2=0} = -ieQ_f\gamma_\mu; \tag{6}$$

On-Shell Renormalization Conditions

3. For fermion self energy:

$$\lim_{\not{k}\to m} \frac{\hat{\Sigma}_{ff}(\not{k})}{\not{k}-m} u(k) = 0; \tag{7}$$

which leads to

$$\hat{\Sigma}_{ff}(\not{k}=m) = 0; \tag{8}$$

and using L'Hospital's rule,

$$\left.\frac{\partial \hat{\Sigma}_{ff}(\not{k})}{\partial \not{k}}\right|_{\not{k}=m} = 0 \tag{9}$$

Multiplicative Scheme

The multiplicative scheme, or des Cloizeaux' scheme, introduces scalar terms to re-size the parameters, fermion field ψ, electric charge e, mass m, and boson field A_μ. After re-scaling all of the parameters, the Lagrangian will be renormalized as a result.

Using $z_i = 1 + \delta z_i$ $(i = e, m, \psi, \gamma)$, the scaled terms can be written as

$$e_0 \rightarrow z_e e = (1 + \delta z_e)e;$$

$$m_0 \rightarrow z_m m = (1 + \delta z_m)m;$$

$$\psi_0 \rightarrow \sqrt{z_\psi}\psi \approx (1 + \frac{1}{2}\delta z_\psi)\psi;$$

and

$$A_\mu^0 \rightarrow \sqrt{z_\gamma}A_\mu \approx (1 + \frac{1}{2}\delta z_\gamma)A_\mu;$$

where $\delta z_\psi, \delta z_e, \delta z_m, \delta z_\gamma$ are undetermined constants.

Multiplicative Scheme

Since the multiplicative scheme is a systematic approach, it applies to all the renormalizable fields. However, it may not be the most efficient approach. Therefore, this demonstration is only for QED at the one-loop level and no weak interactions are involved.

First of all, we need to obtain the renormalized Lagrangian term for the interaction of the fields. The original Lagrangian for QED is

$$L_{QED}^{0} = \bar{\psi}_0 (i\partial\!\!\!/ - e_0 A\!\!\!/_0 - m_0)\psi_0 - \frac{1}{4} F_{\mu\nu}^{0} F_0^{\mu\nu}, \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The renormalized Lagrangian for QED becomes

$$\begin{aligned}\hat{L}_{QED} =& (1+\delta z_\psi)\bar{\psi}(i\partial\!\!\!/ - (1+\delta z_e)e\left(1+\frac{1}{2}\delta z_\gamma\right)A\!\!\!/ - (1+\delta z_m))\psi - \\ &- \frac{1}{4}\left(1+\frac{1}{2}\delta z_\gamma\right)^2 F_{\mu\nu}F^{\mu\nu} \\ =& \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \delta z_\psi \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi + \bar{\psi}(-\delta z_e)eA\!\!\!/\psi + \\ &+ \bar{\psi}\left(-\frac{1}{2}\delta z_\gamma\right)eA\!\!\!/\psi + \bar{\psi}(-\delta z_m)m\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\delta z_\gamma\end{aligned}$$

Multiplicative Scheme

The additional terms in $\hat{L}_{QED}$ are known as the counterterm Lagrangian from which we can derive the counterterms.
Vertex coupling:

$$\hat{\Gamma}_\mu = \Gamma_\mu + \delta\Gamma_\mu,$$
$$\delta\Gamma_\mu = -ieQ_f\gamma_\mu\left(\delta z_e + \frac{1}{2}\delta z_\gamma + \delta z_\psi\right). \quad (12)$$

Fermion self energy loop:

$$\hat{\Sigma}_{ff} = \Sigma_{ff} + \delta\Sigma_{ff},$$
$$\delta\Sigma_{ff} = \not{k}\delta z_\psi - m(\delta z_\psi + \delta z_m). \quad (13)$$

Boson self energy loop:

$$\hat{\Sigma}_{\gamma\gamma} = \Sigma_{\gamma\gamma} + k^2\delta\Sigma_{\gamma\gamma},$$
$$\delta\Sigma_{\gamma\gamma} = -\delta z_\gamma. \quad (14)$$

Now we need to solve all the renormalization constants δz_i from the renormalization conditions.

Multiplicative Scheme

Now the renormalized graphs with the corresponding counterterms are
Vertex:

$$\hat{\Gamma}_\mu(q^2) = \Gamma_\mu(q^2) - ie\gamma_\mu Q_f F_1(0) = \Gamma_\mu(q^2) - \Gamma_\mu(0), \tag{15}$$

Boson self energy loop:

$$\begin{aligned} \hat{\Sigma}_{\gamma\gamma} &= \Sigma_{\gamma\gamma} + \delta\Sigma_{\gamma\gamma} \\ &= \Sigma_{\gamma\gamma} - k^2 \frac{\partial \Sigma_{\gamma\gamma}}{\partial k^2}\bigg|_{k^2=0} ; \end{aligned} \tag{16}$$

Fermion self energy loop:

$$\begin{aligned} \hat{\Sigma}_{ff}(\not{k}) &= \Sigma_{ff}(\not{k}) + \not{k}\delta z_\psi - m(\delta z_\psi + \delta z_m) \\ &= \Sigma_{ff}(\not{k}) - \frac{\partial \Sigma_{ff}(\not{k})}{\partial \not{k}}(\not{k} - m)\bigg|_{\not{k}=m} - \Sigma_{ff}(\not{k})|_{\not{k}=m} . \end{aligned} \tag{17}$$

Mathematica

The numerical calculation approach are mostly done in `Mathematica`. The packages we use, are `FeynArts`, `FormCalc`, `FeynCalc` and `Looptools`. In addition to `Mathematica`, we used Python to convert the leptonic cross section into leptonic tensors.

- FeynArts: Generating Feynman diagrams and amplitude;
- FormCalc and Form: Generating analytical expressions for leptonic amplitude;
- LoopTools: Integration tool.

T. Hahn, Comput. Phys. Commun., 140 418 (2001);
T. Hahn, M. Perez-Victoria, Comput. Phys. Commun., 118, 153 (1999);
J. Vermaseren, arXiv:math-ph/0010025, (2000);
V. Shtabovenko, R. Mertig and F. Orellana, arXiv:1601.01167 (2016);
R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345-359 (1991).

Leptonic Tensor Calculation

To calculate the leptonic tensors, one can calculate the cross section of a truncated $\gamma + p \rightarrow l^+ + l^- + p$,. This simplifies the calculation significantly since it can use the automated techniques to generate cross sections on `Mathematica` .

Further more, one can directly apply the existing regularization and renormalization scheme from regular two-to-two scattering process.

After obtaining the expression for the leptonic cross section, one can use the polarization vectors for the photons to construct the leptonic tensor structure.

Leptonic Tensor Calculation

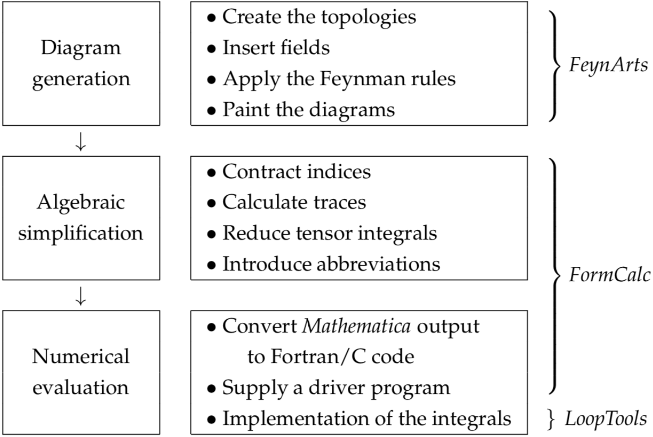


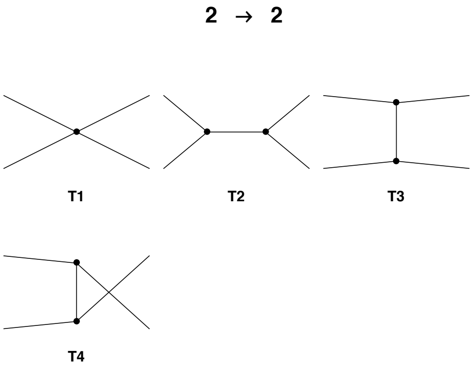
Figure: T. Hahn, "Feynman Diagram Calculations with FeynArts, FormCalc, and LoopTools," PoS ACAT **2010**, 078 (2010) doi:10.22323/1.093.0078

FeynArts

The main functions of `FeynArts` is to define topologies of the process, then construct the Feynman diagrams based on the topologies and the corresponding amplitudes using Feynman rules.

FeynArts

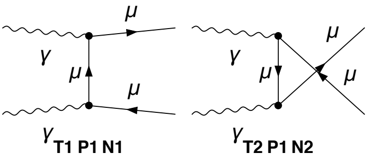
```
t1tree := CreateTopologies[0, 2 → 2]
```



FeynArts

```
t2treeG := InsertFields[t1tree,
    {V[1], V[1]} → {F[2, {2}], -F[2, {2}]}, InsertionLevel → {Particles},
    ExcludeParticles → {S, V[2], V[3]}, Model → "SM", GenericModel → "Lorentz"];
```

$$\gamma \quad \gamma \;\rightarrow\; \mu \quad \mu$$



FeynArts

```
In[∘]:= ampTree = CreateFeynAmp[t2treeG]

Out[∘]= FeynAmpList[Process → {{V[1], p1, 0, {}}, {V[1], p2, 0, {}}} →
          {{F[2, {2}], k1, MM, {-Charge, LeptonNumber}},
           {-F[2, {2}], k2, MM, {Charge, -LeptonNumber}}},
        Model → {SM}, GenericModel → {Lorentz}, AmplitudeLevel → {Particles},
        ExcludeParticles → {S, S[1], S[2], -S[3], S[3], V[2], -V[3], V[3]},
        ExcludeFieldPoints → {}, LastSelections → {}][
       FeynAmp[GraphID[Topology == 1, Generic == 1, Particles == 1, Number == 1],
        Integral[], -ū[k1, MM].(ⅈ EL ga[Lor1].om₋ + ⅈ EL ga[Lor1].om₊).
            (MM + gs[p2 - k2]).(ⅈ EL ga[Lor2].om₋ + ⅈ EL ga[Lor2].om₊).v[k2, MM]
         ep[V[1], p1, Lor1] ep[V[1], p2, Lor2] 1/(-MM2 + (-(p2) + k2)^2)],
       FeynAmp[GraphID[Topology == 2, Generic == 1, Particles == 1, Number == 2],
        Integral[], -ū[k1, MM].(ⅈ EL ga[Lor2].om₋ + ⅈ EL ga[Lor2].om₊).
            (MM + gs[-(p2) + k1]).(ⅈ EL ga[Lor1].om₋ + ⅈ EL ga[Lor1].om₊).v[k2, MM]
         ep[V[1], p1, Lor1] ep[V[1], p2, Lor2] 1/(-MM2 + (p2 - k1)^2)]]
```

FormCalc

After obtaining the expression of the scattering amplitude, one can use FormCalc to construct the desired cross sections and express the analytical results in of Passarino–Veltman basis.

The main advantage of FormCalc is that it provides the fast speed of Form and keep the user friendly interface of Mathematica.

FormClac

```
stuffTree = CalcFeynAmp[ampTree] //. Subexpr[] //. Abbr[]
```

$$4\,\mathrm{MM}\,\pi\,\alpha\left(\frac{1}{\mathrm{MM2}-k[2]+k[3]}-\frac{1}{\mathrm{MM2}+k[2]-k[4]}\right)\mathrm{Mat}[\langle u3|1,e[1],e[2]|v4\rangle]+$$
$$4\,\pi\,\alpha\left(\frac{1}{\mathrm{MM2}-k[2]+k[3]}+\frac{1}{\mathrm{MM2}+k[2]-k[4]}\right)\mathrm{Mat}[\langle u3|1,e[1],e[2],k[2]|v4\rangle]-$$
$$\frac{4\,\pi\,\alpha\,\mathrm{Mat}[\langle u3|1,e[1],e[2],k[3]|v4\rangle]}{\mathrm{MM2}-k[2]+k[3]}-\frac{4\,\pi\,\alpha\,\mathrm{Mat}[\langle u3|1,e[1],e[2],k[4]|v4\rangle]}{\mathrm{MM2}+k[2]-k[4]}-$$
$$\frac{8\,\mathrm{MM}\,\pi\,\alpha\,\mathrm{Mat}[\langle u3|1|v4\rangle]\,\mathrm{Pair}[e[1],e[2]]}{\mathrm{MM2}-k[2]+k[3]}-\frac{8\,\pi\,\alpha\,\mathrm{Mat}[\langle u3|1,k[2]|v4\rangle]\,\mathrm{Pair}[e[1],e[2]]}{\mathrm{MM2}-k[2]+k[3]}+$$
$$\frac{8\,\pi\,\alpha\,\mathrm{Mat}[\langle u3|1,k[3]|v4\rangle]\,\mathrm{Pair}[e[1],e[2]]}{\mathrm{MM2}-k[2]+k[3]}-$$
$$(8\,\pi\,\alpha\,\mathrm{Mat}[\langle u3|1,e[2]|v4\rangle]\,(\mathrm{Pair}[e[1],k[1]]-\mathrm{Pair}[e[1],k[4]]))\,/$$
$$(\mathrm{MM2}-k[2]+k[3])+$$
$$(8\,\pi\,\alpha\,\mathrm{Mat}[\langle u3|1,e[1]|v4\rangle]\,(\mathrm{Pair}[e[2],k[1]]-\mathrm{Pair}[e[2],k[3]]))\,/$$
$$(\mathrm{MM2}+k[2]-k[4])$$

FormClac

```
Hel[_] = 0;
Heltree = HelicityME[stuffTree, stuffTree];
squareTree = SquaredME[stuffTree, stuffTree] //. Heltree //. Subexpr[] //. Abbr[]
tree = squareTree[[1]] //. squareTree[[2]]
```

$$-\frac{1}{m2 - k[2] + k[3]} 8\, MM\, \pi\, \alpha\, \mathrm{Pair}[e[1], e[2]] \left(4\, MM\, \pi\, \alpha^* \left(\frac{1}{m2 - k[2] + k[3]} - \frac{1}{m2 + k[2] - k[4]}\right) \mathrm{Mat}[\langle u3|1|v4\rangle, \langle u3|1,e[1],e[2]|v4\rangle] + 4\, \pi\, \alpha^* \left(\frac{1}{m2 - k[2] + k[3]} + \frac{1}{m2 + k[2] - k[4]}\right.\right.$$

$$\mathrm{Mat}[\langle u3|1|v4\rangle, \langle u3|1,e[1],e[2],k[2]|v4\rangle] - \frac{4\, \pi\, \alpha^*\, \mathrm{Mat}[\langle u3|1|v4\rangle, \langle u3|1,e[1],e[2],k[3]|v4\rangle]}{m2 - k[2] + k[3]} - \frac{4\, \pi\, \alpha^*\, \mathrm{Mat}[\langle u3|1|v4\rangle, \langle u3|1,e[1],e[2],k[4]|v4\rangle]}{m2 + k[2] - k[4]} - \cdots$$

LoopTools

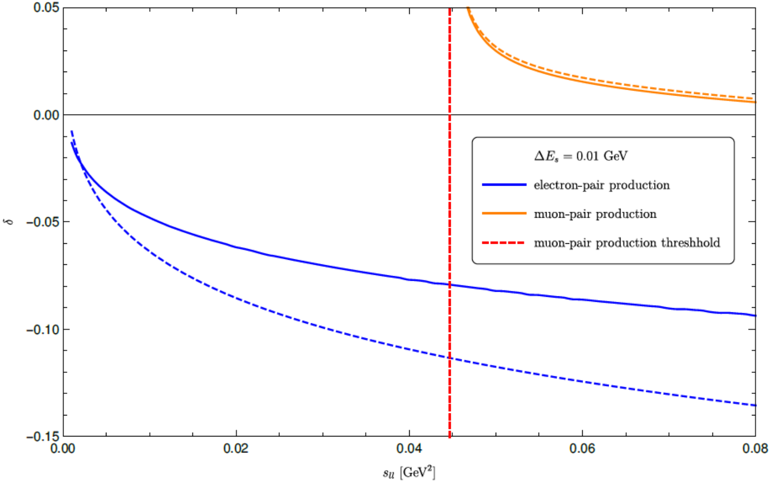
LoopTools is a numerical integration package to evaluate the one-loop-level Passarino-Veltman functions, which produces the final result from FeynCalc output.

Tensor Contraction

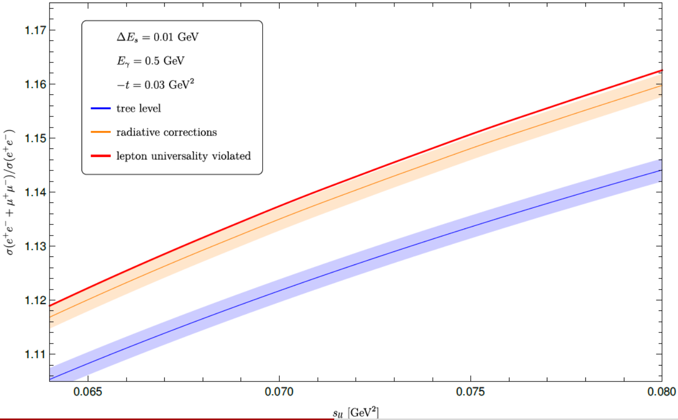
After obtaining the expression for the leptonic cross section, one will use the polarization vectors for the photons to construct the leptonic tensor. We created a python script to replace all the photon polarization vectors into tensorial form in the format of FeynCalc notation.

Note that after the contraction with the hadronic tensor, the scalar result would still be infrared divergent. One has to include the soft photon bremsstrahlung to cancel the IR divergence.

Comparison of first-order QED corrections contribution δ to the cross section, which is defined as $d\sigma/ds_{ll} \equiv (1+\delta)(d\sigma/ds_{ll})_0$. It includes soft-photon Bremsstrahlung with $\Delta E_s = 0.01$ GeV (solid lines), with the calculation in the soft-photon approximation (dashed lines). The vertical dashed red line indicates the muon-pair production threshold at $s_{ll} \approx 0.0045$ GeV^2.



Ratio of cross sections between electron- and muon-pair production at tree level (blue curve) and with account of first-order QED corrections estimated using $\Delta E_s = 0.01$ GeV (orange curve) with 3σ error bands. The red curve denotes the scenario when lepton universality is broken with G^{μ}_{Ep}/G^{e}_{Ep} =1.01, including the full set of next-to-leading-order radiative corrections. The 3σ significance level, if one is able to measure the ratio of the cross sections with an absolute precision of around 7×10^{-4}. An upcoming experiment at MAMI is planned to perform such measurements [7].



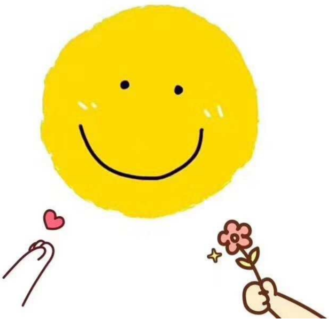
Acknowledgment

Many thanks to

- Grenfell Campus of Memorial University of Newfoundland
- NSERC

Acknowledgement

I would like to thank both of my advisors – Dr. Aleksejevs and Dr. Barkanova; Matthias Heller, Oleksandr Tomalak and their advisor Dr. Marc Vanderhaeghen for the collaboration.

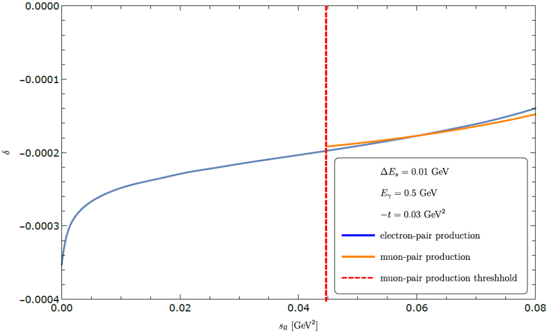


References

- M. Heller, O. Tomalak and M. Vanderhaeghen, Phys. Rev. D **97**, no. 7, 076012 (2018) doi:10.1103/PhysRevD.97.076012 [arXiv:1802.07174 [hep-ph]].
- J. Arrington, J. Phys. Chem. Ref. Data **44**, 031203 (2015) doi:10.1063/1.4922414 [arXiv:1506.00873 [nucl-ex]].
- T. Hahn, PoS ACAT **2010**, 078 (2010) doi:10.22323/1.093.0078 [arXiv:1006.2231 [hep-ph]].
- J. C. Bernauer *et al.* [A1 Collaboration], Phys. Rev. Lett. **105**, 242001 (2010) doi:10.1103/PhysRevLett.105.242001 [arXiv:1007.5076 [nucl-ex]].
- A. Antognini *et al.*, Science **339**, 417 (2013). doi:10.1126/science.1230016
- I. T. Lorenz and U. G. Meißner, Phys. Lett. B **737**, 57 (2014) doi:10.1016/j.physletb.2014.08.010 [arXiv:1406.2962 [hep-ph]].
- A2 Coll. at MAMI, Letter of intent, `https://www.blogs.uni-mainz.de/fb08-mami-experiments/files/2016/07/A2-LOI2016-1.pdf`

Results

First-order QED cross-section correction on a proton side in the soft-photon limit, with $\Delta E_s = 0.01$ GeV. The vertical dashed red line indicates the muon-pair production threshold at $s_{ll} \approx 0.0045$ GeV^2



Results

First-order QED cross-section correction on a proton side in the soft-photon limit as a function of t, with $\Delta E_s = 0.01$ GeV.