The Fine Structure Constant up to Two-Loop Level

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Overview

- Motivation for two-loop calculations
- Precision frontier
- Running of the fine structure constant
- One-loop result
- Dispersion approach
- Renormalization in dispersion representation
- Results for two-loop (ongoing)
- Conclusion

Motivation behind two-loop calculations

- Standard Model of Particle Physics does not explain everything.
- Few examples :
  - Fundamental symmetry
  - Matter - Antimatter asymmetry
  - Hierarchy problem
  - Supersymmetry
  - Dark Matter, Dark energy ...
- We can explore in 3 different paths:
  - The Energy Frontier
  - The Cosmic Frontier
  - The Precision Frontier

Our work is on Precision Frontier

Precision frontier

- It provides an alternative approach to probe physics beyond the Standard model.
- Extremely rare processes with tiny deviation from existing models can be explained.
- General approach is to consider higher order Feynman diagrams to give us a more precise theoretical predictions at sub-percent level.
- For instance, in scattering processes, the perturbation expansion of the scattering matrix could leads us to sub-percent level accuracy.
- New virtual particles in the form of a mediator between interactions can be modelled.

Running of the fine structure constant

- Fine structure constant, α is very accurately determined at zero momentum transfer.
- QED vacuum polarization corrections result in a Q^2 dependence of the effective fine-structure constant which is usually parameterized as:

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(Q^2)} = \frac{\alpha(0)}{1 - \hat{\Pi}(Q^2)}$$

  where $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$ is the renormalized vacuum polarization function.
- The number on right hand side of the equation not only depends on the the tree level interaction but also on other particles that could appear as virtual particles in the one-loop, two-loop and so on.

Feynman diagrams: one-loop

With the help of FeynArts we have generated all one-loop Feynman diagrams:

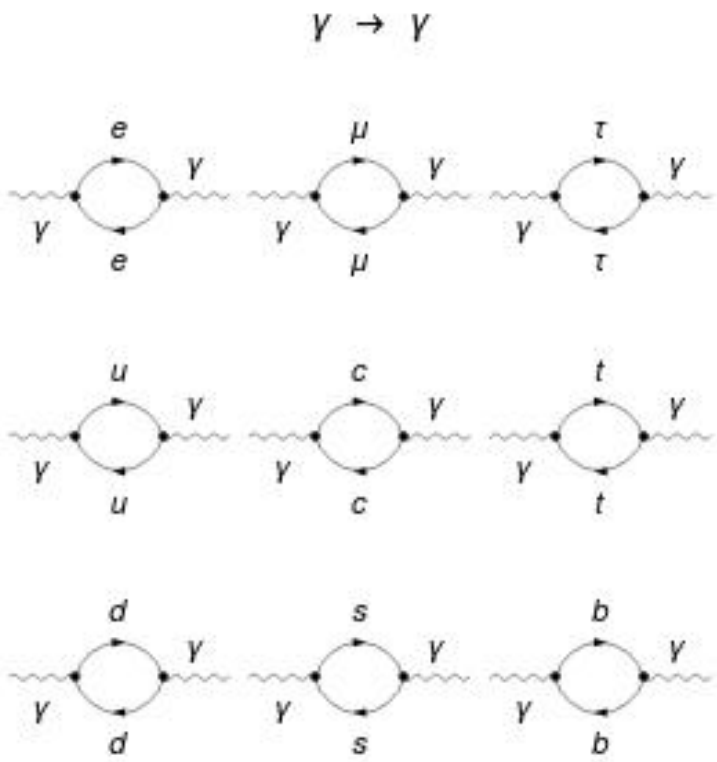


Figure: One-loop Feynman diagrams

Apart from the differences in mass of the quarks and leptons, there is only one type of topology.

Self-energy: one-loop

- As an example, I chose the following topology:

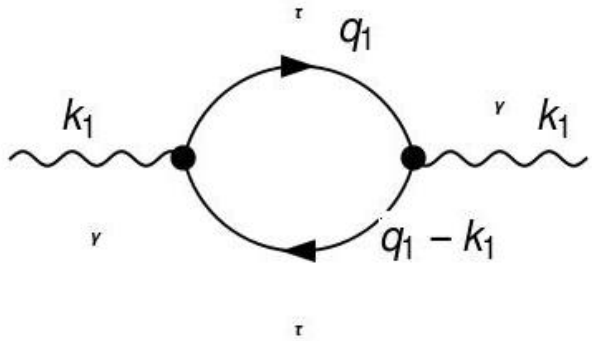


Figure: One-loop self energy diagram

- The general structure of the amplitude of this topology is:

$$\hat{\Pi}_{\mu\nu}(k_1^2) = i\left(g_{\mu\nu} - \frac{k_{1\mu}k_{1\nu}}{k_1^2}\right) k_1^2 \hat{\Pi}(k_1^2)$$

where $\hat{\Pi}(k_1^2)$ is the renormalized vacuum polarization function is given $\hat{\Pi}(k_1^2) = \Pi(k_1^2) - \Pi(0)$ which gives

$$\alpha(k_1^2) = \frac{\alpha(0)}{1 - \hat{\Pi}_{1L}(k_1^2)}$$

Running of the fine structure constant up to one-loop level

- I have used FormCalc and LoopTools to get the numerical result.
- Effective quark mass is chosen for this calculation.
- The table shows the corrections at one loop level

$k_1{}^2/\mathrm{GeV}^2$	Effective Fine Structure Constant	Correction at one-loop
0.001	0.00733420729459514	0.0000368542149503165
0.1	0.0073663549017348	0.0000690018220899837
10.	0.00746874026798723	0.000171387188342406
1000	0.00766546562472932	0.0003681125450845
100 000	0.00785171297808687	0.00055435989844204 8
1000 000	0.00797896797610224	0.000681614896457417

Figure: Table showing the difference in effective fine structure constant and its correction at one-loop correction

Running of the fine structure constant upto one-loop level

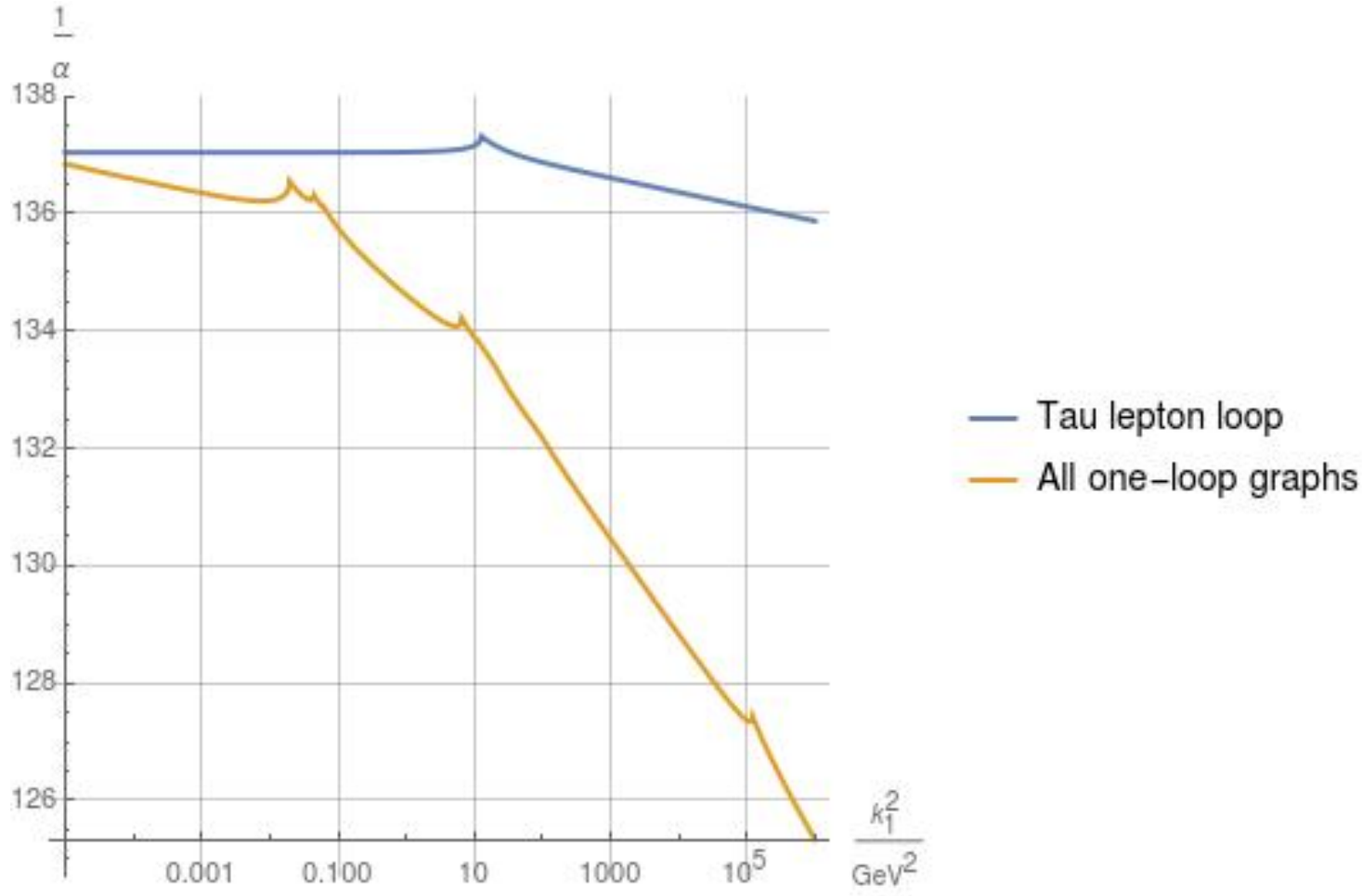
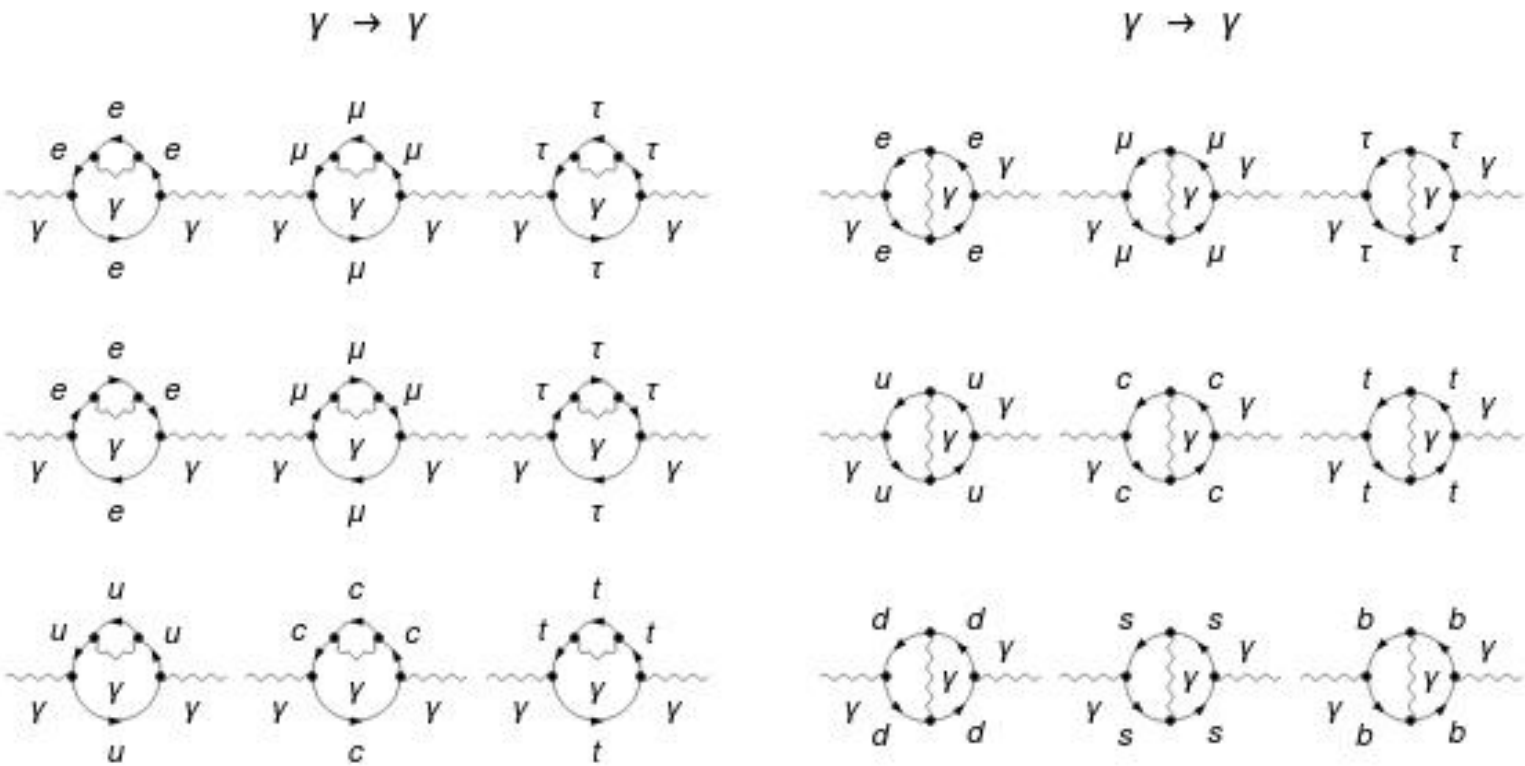


Figure: Running of the fine structure constant up to one-loop

Feynman diagrams: two-loop

With the help of FeynArts we generated the two-loop diagrams:



Apart from the difference in masses, there are only *two* types of topology here.

Self-energy: two-loop

- One-loop level calculation can easily be done using computer packages like FeynArts, FormCalc, etc.
- Our main focus is on two-loop self- energy diagram.
- As an example we chose the following topology:

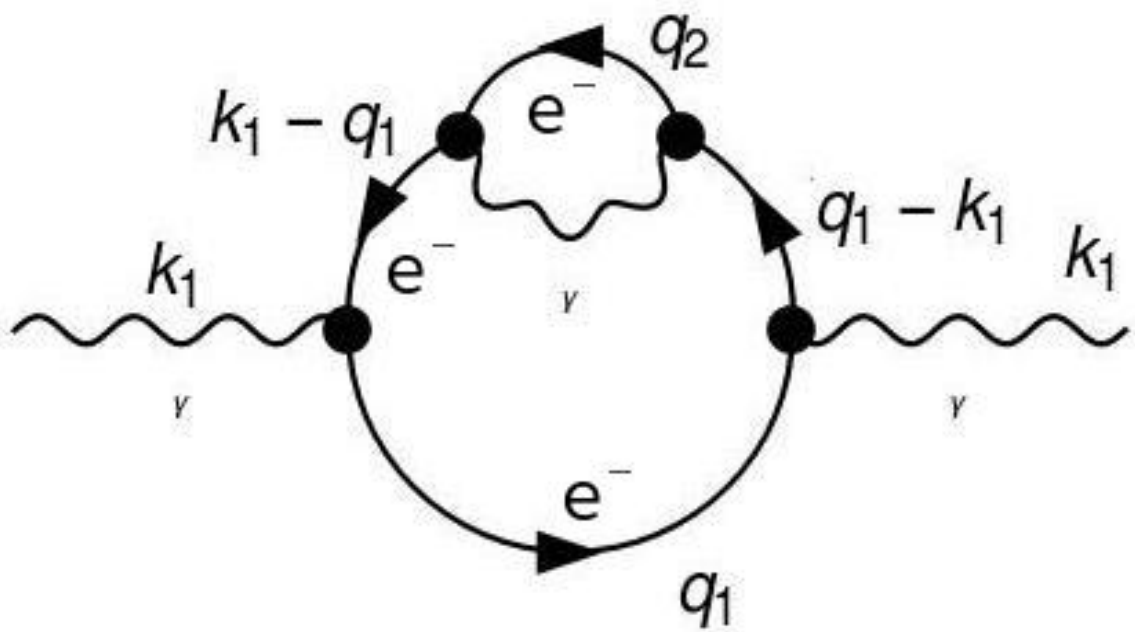


Figure: Two-loop level self-energy diagram

Amplitude of the two-loop self-energy

- The amplitude of this diagram has the following structure:

$$\begin{aligned}\Pi_{\mu\nu} &= \left(g_{\mu\nu} - \frac{k_{1\mu}k_{1\nu}}{k_1^2}\right) k_1^2 \Pi(k_1^2) \\ &= \int d^4q_1 d^4q_2 \\ &\times \frac{\mathfrak{D}_{\mu\nu}}{(q_1^2 - m_l^2)(q_2^2 - m_l^2)\left((q_1 - k_1)^2 - m_l^2\right)} \\ &\times \frac{1}{\left((q_1 - k_1)^2 - m_l^2\right)(q_2 + k_1 - q_1)^2}\end{aligned}$$

where

$$\begin{aligned}\mathfrak{D}_{\mu\nu} = &Tr\left[(m_l + \not{k}_1 - \not{q}_1)(ie\gamma^\nu)(m_l + \not{q}_1)\right] \\ &\times Tr\left[(ie\gamma^\mu)(m_l + \not{q}_1 - \not{k}_1)(ie\gamma^\sigma)(m_l + \not{q}_2)(ie\gamma^\rho)\right] \\ &\times g_{\rho\sigma}\end{aligned}$$

Amplitude of the two-loop self-energy

- Our aim is to evaluate this amplitude and see two-loop contribution in

$$\alpha(k_1^2) = \frac{\alpha(0)}{1 - \left(\hat{\Pi}_{1L}(k_1^2) + \hat{\Pi}_{2L}(k_1^2)\right)}$$

- Because of different mass propagators and higher order tensors, it is sometimes impossible to find analytical results for this kind of two loop topology which leads us to use different numerical methods.
- We chose to use the Dispersion approach.
- In dispersion approach, a sub-loop can be represented through a dispersion tensor integral operator with a simple propagator-like structure and the dispersion tensor integral can be absorbed into the effective Feynman propagators in the second loop integral.

Dispersion approach

- In short, any sub-loop insertion of the form:

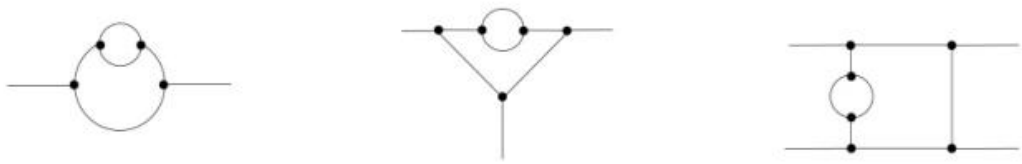


Figure: General topologies with sub-loop

can be reduced using the Dispersion technique.

- So, we will be dealing with reduced topologies of the form:

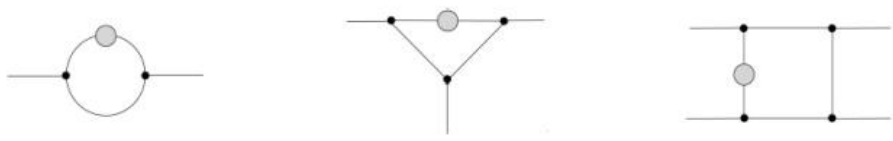


Figure: Reduced topologies with no sub-loop

- Consequently, we will have a simpler analytical structure and calculations will be much faster.

Dispersion approach

- A general topology like this:

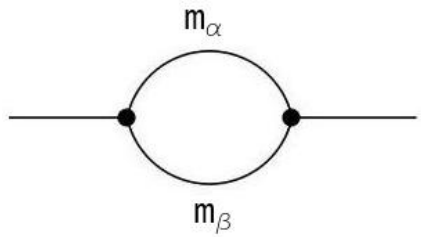


Figure: A general self-energy topology

with two-point tensor coefficient functions (in Passarino-Veltman basis) can be replaced as follows:

$$B_{i,ij,ijk}(q^2, m_\alpha^2, m_\beta^2) = \frac{1}{\pi} \int_{(m_\alpha + m_\beta)^2}^{\infty} ds \frac{\Im[B_{i,ij,ijk}(s^2, m_\alpha^2, m_\beta^2)]}{s - q^2 - i\epsilon}$$

with a dispersion tensor.

Our approach

- To use the dispersion we have to first evaluate the insertion as shown below:

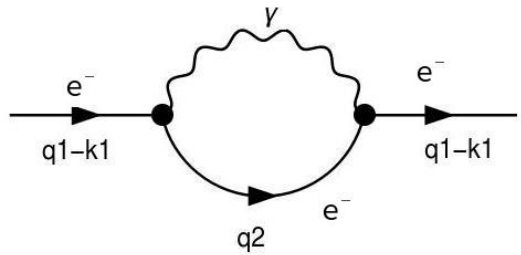


Figure: Two-loop level self-energy diagram

- The insertion will then be replaced and which will simplify the amplitude as:

$$\int d^4 q_1 ds \frac{\mathfrak{D}_{\mu\nu}}{(q_1^2 - m_l^2)\left((q_1 - k_1)^2 - m_l^2\right)} \frac{1}{(s - (q_1 - k_1)^2 - i\epsilon)}$$

- One of the propagator, $(q_1 - k_1)^2 - m_l^2$ gets cancelled with the numerator of the dispersion tensor.
- Effectively leaving us with three propagators only.

Renormalization of self-energy in QED

In general a fermion self-energy graph with a amplitude structure

$$\hat{\Sigma}(\not{k}) = \not{k}\hat{\Sigma}_V(k^2) + \hat{\Sigma}_S(k^2)$$

is renormalized as follows:

$$\hat{\Sigma}_V(k^2) = \Sigma_V(k^2) - \Sigma_V(m^2) - 2m^2\left(\Sigma'_V(m^2) + \Sigma'_S(m^2)\right)$$
$$\hat{\Sigma}_S(k^2) = m\Sigma_S(k^2) - m\Sigma_V(m^2) + 2m^3\left(\Sigma'_V(m^2) + \Sigma'_S(m^2)\right)$$

where $\hat{\Sigma}_V$ and $\hat{\Sigma}_S$ are the renormalized amplitude and

$$\Sigma'_V = \frac{\partial \Sigma_V}{\partial k^2}|_{k^2=m^2}$$
$$\Sigma'_S = \frac{\partial \Sigma_S}{\partial k^2}|_{k^2=m^2}$$

The last part of these equations are the UV-Finite part of the renormalized amplitude.

Renormalization in dispersion approach

Similarly, in the dispersive representation of the amplitude we get the following renormalized amplitude:

$$\hat{\Sigma}_V(k^2) = \frac{k^2 - m^2}{\pi} \int_{m^2}^{\infty} ds \frac{Im\left(\Sigma_V(s)\right)}{(s - k^2)(s - m^2)} - 2m^2 F$$

$$\hat{\Sigma}_S(k^2) = \frac{m(k^2 - m^2)}{\pi} \int_{m^2}^{\infty} ds \frac{Im\left(\Sigma_S(s)\right)}{(s - k^2)(s - m^2)} + 2m^3 F$$

where

$$F = \int_{m^2}^{\infty} \frac{Im\left(\Sigma_V(s) + \Sigma_S(s)\right)}{(s - m^2)^2}$$

F are convergent integrals and are constants.

Renormalization in dispersion approach

- Next we evaluate the renormalized vacuum polarization tensor

$$\hat{\Pi}_{\mu\nu} = \hat{\Pi}_{2L} = \Pi(q^2) - \Pi(0)$$

we can use the relationship

$$\alpha(k_1^2) = \frac{\alpha(0)}{1 - \left(\hat{\Pi}_{1L}(k_1^2) + \hat{\Pi}_{2L}(k_1^2)\right)}$$

to evaluate the running of the fine structure constant.

- So far, I have only done one graph at two-loop level, as an example.

Resuts

- In my example, I chose the following graph:

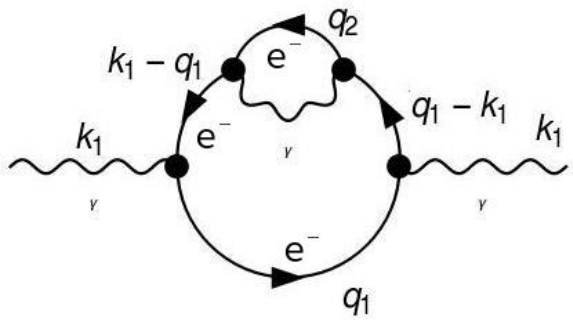


Figure: Two-loop level self-energy diagram

- I have calculated the UV-Finite part seperately for this and it has no contribution.
- All the contribution is from the part which was represented by Dispersion tensor.

Resuts

The table shows the corrections at two loop level with only one graph of electron considered.

$k_1{}^2/\mathrm{GeV}^2$	Effective Fine Structure Constant$(e^-\text{-graph})$	Correction at two-loop
0.001	0.0073412437655255	$8.29180425900716 \times 10^{-8}$
0.1	0.0073622291410389	$1.31987630913252 \times 10^{-7}$
10.	0.0074685548659385 1	$1.85402048710932 \times 10^{-7}$
1000	0.0076652451677067 4	$2.20457022579064 \times 10^{-7}$
100 000	0.0078514812352501 1	$2.31742836757226 \times 10^{-7}$
1 000 000	0.0079787286565005	$2.39319601736537 \times 10^{-7}$

Figure: Table showing the difference in effective fine structure constant and its correction at two-loop correction (only electron graph)

Conclusion

- Currently I am only considering Quantum Electrodynamics (QED) but this can be extended to cover Electroweak sectors and will help us to evaluate running of the Weinberg mixing angle upto two-loop level.
- The dispersion representation also works for triangle and box topology which will be my next calculation.
- In future I will try to develop an extension for the FormCalc that will be able to do all the manual insertions of the dispersion integral automatically.
- In addition, I will extensively use the dispersion approach to reduce the current calculation time of existing computer packages.

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